Tamworks

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1. Introduction

2. Cooking Meatballs

Problem 1. *A swordfish meatball, which can be approximated as a sphere with a diameter of 4 cm, is taken directly from the refrigerator at a uniform temperature of 5°C and placed in a convection oven set at 375°F (190.6°C). The goal is to determine how long it will take for the center of the meatball to reach 60°C, which is the desired internal temperature for perfect cooking. (I gave it a sear for texture but it doesn not significantly change the internal temp by much. Furthermore, I've also taken into account the thin layer of puff pastry)*

Known Parameters

- *Initial temperature of the meatball:* $T_i = 5°C$
- *Oven temperature:* $T_\infty = 190.6°C$
- *Desired internal temperature (center):* $T_{center} = 60°C$
- *Convection heat transfer coefficient:* $h = 100\ W/m^2 \cdot K$
- *Meatball radius:* $r_o = 0.02\, m$ *(4 cm diameter)*
- *Thermal conductivity of swordfish meat:* $k = 0.6\ W/m \cdot K$
- *Thermal diffusivity of swordfish meat:* $\alpha = 1.8 \times 10^{-7}\ m^2/s$

Step 1: Biot Number Calculation

First, we calculate the Biot number (Bi) to determine whether we can apply the lumped capacitance model. The Biot number is given by:

$$Bi = \frac{hr_o}{k}$$

Substituting the known values:

$$Bi = \frac{100 \times 0.02}{0.6} = 3.33$$

Since $Bi > 0.1$*, the lumped capacitance model is not applicable, so we will use the one-term approximation for transient heat conduction in a sphere.*

Step 2: One-Term Approximation Constants

For a Biot number of 3.33*, we use the constants from the Heisler charts:*

$$\lambda_1 = 1.8, \quad A_1 = 1.2$$

Step 3: Transient Heat Conduction Equation

We will now use the one-term approximation for transient heat conduction in a sphere. The equation is:

$$\frac{T(0,t) - T_\infty}{T_i - T_\infty} = A_1 \cdot e^{-\lambda_1^2 \frac{\alpha t}{r_o^2}}$$

Where:

- $T(0,t)$ *is the center temperature at time* t*,*
- $T_i = 5°C$ *is the initial temperature of the meatball,*
- $T_\infty = 190.6°C$ *is the oven temperature,*
- $r_o = 0.02\,m$ *is the radius of the meatball,*
- $\alpha = 1.8 \times 10^{-7}\,m^2/s$ *is the thermal diffusivity.*

Step 4: Substituting the Known Values

Substituting the known values into the equation:

$$\frac{60 - 190.6}{5 - 190.6} = 1.2 \cdot e^{-(1.8)^2 \frac{1.8 \times 10^{-7} \cdot t}{(0.02)^2}}$$

Simplifying the left-hand side:

$$\frac{-130.6}{-185.6} \approx 0.704$$

Thus, the equation becomes:

$$0.704 = 1.2 \cdot e^{-9.72 \times 10^{-3} \cdot t}$$

Step 5: Solving for t

We now solve for the cooking time t.

- *Divide both sides by 1.2:*

$$\frac{0.704}{1.2} = e^{-9.72\times10^{-3}\cdot t}$$
$$0.586 = e^{-9.72\times10^{-3}\cdot t}$$

- *Take the natural logarithm of both sides:*

$$\ln(0.586) = -9.72\times10^{-3}\cdot t$$
$$-0.534 = -9.72\times10^{-3}\cdot t$$

- *Solve for t:*

$$t = \frac{0.534}{9.72\times10^{-3}} \approx 28.6\, minutes$$

Final Solution and Comments

*The time required for the center of the swordfish meatball to reach $60°C$ in a convection oven set to $375°F$ (190.6°C) is approximately **28.6 minutes**. I promise you won't overcook the fish!*

Step 3: Solve for the Cooking Time t

Finally, I solve this equation to determine t, taking for the center of the swordfish meatball to reach 60°C.

3. Volume and Distribution

Problem 2. *In this recipe, I had the wonderful idea to make a sweet potato tiered cake, but you only have $C inches^3$. I want it such for each layer of cake, it's height will be 1 inch and as you go up each tier, it's square side length decreases by 1 inch. Arithmetic progressions in lengths give this sort of balanced look to design; in this case I'm using cakes! You want the cake to have x tiers and you need to find the largest side length, y to start the first cake tier. We can first write an expression representing the volume of each layer of cake summing up to C inches ^{3}.*

We are calculating the total volume of the cake. Let the number of tiers be n, where $x = n - 1$, and the smallest tier has dimensions y.

I was able to find the volume of sweet potato batter to be perfectly $600cm^3$, whichis the sum of the volumes of the tiers:

$$600 = \sum_{i=0}^{x}(y+i)^2(1)$$

This expands to:

$$600 = \frac{(y+x)(4y+2x+1)(2y+2x+1)-(y-1)(y)(2y-1)}{6}$$

Simplifying:

$$3600 = 2x^3 + 6xy^2 + 6xy + 6x^2y + 6y^2 + 3x^2 + x$$

I'm going to add 2 and subtract 2 on both sides of the equation. This makes factoring much easier.

$$3600 + 2 - 2 = (2x^3 + 2) + (6x^2y + 6xy) + (6xy^2 + 6y^2) + (3x^2 + x - 2)$$

Factoring:

$$3600 = (x + 1)(2x^2 - x + 1) + (x + 1)(6xy) + (x + 1)(6y^2) + (3x - 2)(x + 1)$$

We can factor out the $x + 1$, giving us:

$$3600 = (x + 1)(6y^2 + 6xy + 2x^2 + x)$$

At this point, I knew I wanted there to be 3 stacks of sweet potato, which means $x + 1 = 3$ and we can solve for y. This allows me to shape my dough into the correct dimensions before frying.

$$3600 = (3)(6y^2 + 6(2)y + 2(2)^2 + (2))$$

$$y = 13cm$$

And so my tiers ended up being 13cm, 12cm, 11cm, with no batter left. The reason we go through all this trouble doing the algebra is because volume of ingredients end up being really nice number is due to it's near perfect symmetry. It gives us nice composite integers to work with so I'm allowed to choose how many tiers I want. Pretty Neat!

Summary

Using the dimensions of each tier and the total volume formula, you can find the dimensions of the smallest tier and calculate the total volume of the cake.

4. Reduction and Sauces

Problem 3. *Most pans come it a frustum-like shape. In my recipe, I want to reduce some stock into a sauce and I want to know how long it will take to do so. On a simmer, closing in on a boil, water evaporates at the rate of 0.04cm^3 per hour for each cm^2 of exposed surface area of water.*

Knowing how the rate of evaporating water, we can express for every small change of volume, it's a small change in time multiplied by the rate value and surface area:

$$dV = (900)(SurfaceArea)(dt)$$

We can rewrite area in terms of volume so we can integrate a function for time later on. Pans will often come in a fustrum shape. Or more specifically, we want a function for the radius of the pan as it evaporates *since Surface Area is πr^2.(Picture provided is the cross-section of pan if I were to cut down it's diameter). Say you measure the top and bottom radius and depth of the pan, R, r, and h respectively. What I want to do first is find the ratio of the pan's radius at any depth of liquid. To do so, I extend the frustrum cross section into a cone. I'll label this new altitude as l. Now I'll find this unknown variable l in terms of our known constants using similar triangles:*

$$\frac{r}{l} = \frac{R}{h+1} \tag{1}$$

$$l = \frac{rh}{R-r} \tag{2}$$

Now we begin to find an actual function for the radius, x, of the pan at its depth y. Once again, using similar triangles:

$$\frac{\frac{rh}{R-r}}{r} = \frac{\frac{rh}{R-r} + y}{x} \tag{3}$$

$$x = r + \frac{y(R-r)}{h} \tag{4}$$

Next, we want to substitute the variable y with constants and the volume variable, V.

Volume of a Frustum

$$V = \frac{1}{3}\pi y(R^2 + r^2 + Rh) \tag{5}$$

Isolate the y variable from the equation:

$$y = \frac{3V}{\pi(R^2 + r^2 + Rr)} \tag{6}$$

Plug in the equation for y into equation (4)

$$x = r + \frac{(R-r)}{h}\left(\frac{3V}{\pi(x^2 + r^2 + xr)}\right) \tag{7}$$

Multiply both sides by $x^2 + xr + r^2$ and simplify

$$x(x^2 + xr + r^2) - r(x^2 + xr + r^2) = \frac{3V(1(R-r))}{\pi h} \tag{9}$$

$$x^3 + x^2 r + xr^2 - r^3 - r^2 x - r^2 r = \frac{3V(R-r)}{\pi h} \tag{10}$$

$$x^3 = \frac{3V(R-r)}{\pi h} + r^3 \tag{11}$$

$$x^2 = \left(\frac{3V(R-r)}{\pi h} + r^3\right)^{2/3} \tag{12}$$

Now, we can put this equation for radius, x^2, back into our differential equation and integrate to find time:

$$dV = dt(0.04)\left[\frac{V(R-3r)}{\pi h} + \left(\frac{3}{R^3 - r^3}\right)^{2/3}\right]\pi \tag{13}$$

I'm going to plug in the dimensions of my pan where $R = 15$, $r = 12$, $h = 5$. I reduced 50ml of my stock mixture to 25ml of a thickened sauce.

$$\int_{25}^{50} \frac{dx}{\left(x\left(\frac{3(15-12)}{5\pi}\right) + 12^3\right)^{\frac{2}{3}} \cdot 0.12\pi} = 4.5 \textbf{\textit{ minutes}} \tag{13}$$

5. Group Theory and Spices

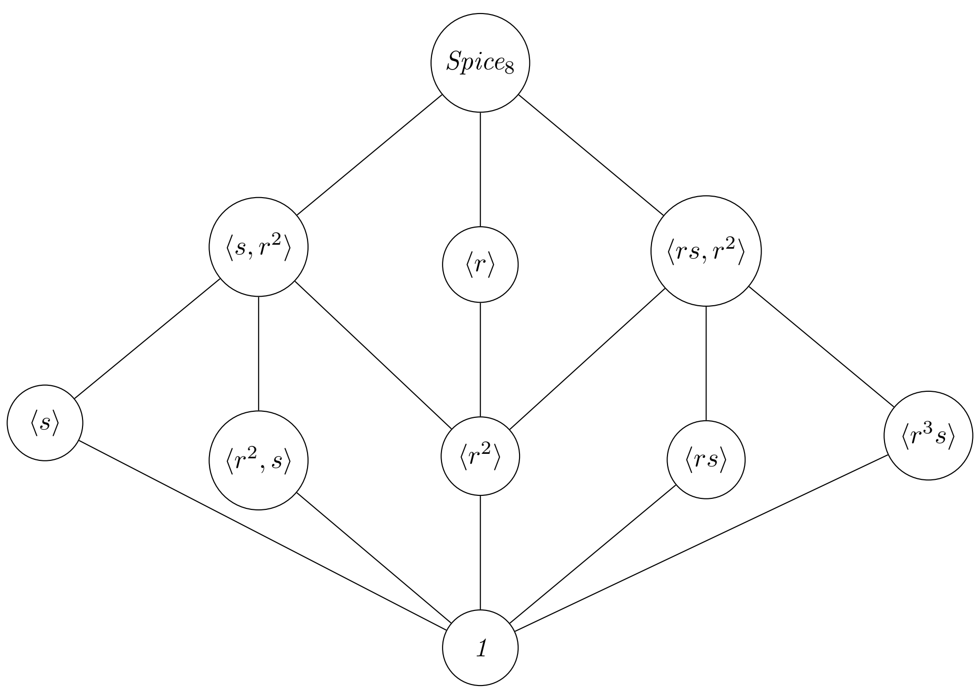
Problem 4. *We are studying the group theory of the **Spice Group**, which is isomorphic to the dihedral group D_8:*

$$Spice_8 \equiv D_8$$

$$\begin{aligned} Spice_8 &= \langle r, s \mid r^4 = s^2 = 1, rs = sr^{-1} \rangle \\ &= \langle r, s \mid r^4, s^2, r^n = 1 \rangle \\ &= \{r, r^2, r^3, sr, s, sr^2, sr^3, 1\} \end{aligned}$$

Lattice Diagram

Below is the lattice diagram for the Spice group:



Spice Definitions

Here are the interpretations of the elements in the Spice group:

$$\begin{aligned}
r &\to \textit{vinegar} \\
r^2 &\to \textit{peppercorns} \\
r^3 &\to \textit{salt} \\
s &\to \textit{sugar} \\
1 &\to \textit{neutral and flavored combinations}
\end{aligned}$$

In this spice model, the identity element 1 *represents a balanced or neutral flavor—one that is either the absence of spice or a perfect equilibrium between contrasting flavors.*

For example, $r^2 = \textit{peppercorns}^2 = 1$ *suggests that peppercorns, when used in a certain proportion, can neutralize their own strong, numbing effect, bringing the flavor profile back to a neutral state.*

Similarly, when vinegar (r*) and salt (*r^3*) are combined in the right proportions, they balance each other to produce a neutral effect, akin to how certain flavor pairings can cancel each other out.*

Flavor Interactions

1. $r^2 = \textit{peppercorns}^2 = 1 = \textit{numbing/neutral aftertaste}$
2. $r(r^3) = \textit{salt}(\textit{vinegar}) = 1$
3. $s^2 = \textit{sugar}^2 = \textit{floral light notes}$
4. $r(r^2) = \textit{peppercorns/vinegar} = \textit{savory taste}$

Similarly, the combination of peppercorns and vinegar through $r(r^2)$ *leads to a savory taste—this suggests that pairing a strong acid (vinegar) with the sharpness of peppercorns creates a complementary, savory result.*

The lattice diagram in the image represents how different subgroups (spice combinations) are structured. At the top of the lattice, $\langle r, s \rangle$ *represents the full set of flavor interactions available from the base spices.*

At the end, I went with a combo of double sugar, vinegar and umami/salt.